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Towards understanding structure of the monopole clusters

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We consider geometrical characteristics of monopole clusters of the lattice $SU(2)$ gludynamics. We argue that the polymer approach to the field theory is an adequate means to describe the monopole clusters. Both finite-size and the infinite, or percolating clusters are considered. We find out that the percolation theory allows to reproduce the observed distribution of the finite-size clusters in their length and radius. Geometrical characteristics of the percolating cluster reflect, in turn, the basic properties of the ground state of a system with a gap.

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I. INTRODUCTION

Explanation of the confinement in terms of the monopole condensation was proposed as early as around the year 1974 [1]. Moreover, the idea is strongly supported by the lattice data [2]. Nevertheless, understanding the lattice data in terms of the continuum theory still represents a challenge. Indeed, generically one usually thinks in terms of a Higgs-type model:

$$S_{eff} = \int d^4x (\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 + V(|\phi|^2)) \qquad (1)$$

where ϕ is a scalar field with a non-zero magnetic charge, $F_{\mu\nu}$ is the field strength tensor constructed on the dual-gluon field B_μ, D_μ is the covariant derivative with respect to the dual gluon. Finally, $V(|\phi|^2)$ is the potential energy ensuring that $\langle\phi\rangle \neq 0$ in the vacuum. There exist detailed fits to the numerical lattice data within such a framework [3]. However, relation of the "effective" fields ϕ, B_μ to the fundamental QCD fields remains unclear.

One of the main problems is the lack of understanding of the nature of the monopole field configurations. In particular, there is seemingly no reason to think about the monopoles as quasi-classical objects, for review see [4]. Under the circumstances, it is natural to exploit the lattice measurements to accumulate first data on, so to say, phenomenology of the monopoles. In fact, definition of the monopoles in the non-Abelian case is not unique and we will concentrate on the monopoles defined within the Maximal Abelian Projection (MAP) in SU(2) lattice gauge model.

A gauge-variant definition of the monopoles would not be of much interest by itself. The basic idea is that there are gauge-invariant objects behind which are detected through the projection. This, gauge-invariant facet is manifested in gauge invariant properties of

the MAP monopoles. In particular, (see [5] and references therein) the three dimensional monopole density ρ_{mon} does not depend on the lattice spacing and is given in the physical units:

$$\rho_{mon} \;=\; 0.65(2)\;\sigma_{SU(2)}^{3/2} \;\approx\; \text{const.}\,, \tag{2}$$

where $\sigma_{SU(2)}$ is the string tension.

An important remark is in order here. While discussing the monopole density one should distinguish between finite-size clusters and the percolating cluster [6, 7]. There is a spectrum of the finite-size clusters, as a function of their length, while the percolating cluster is in a single copy. In other words, the percolating cluster fills in the whole of the lattice and its length is proportional to the volume of the lattice V_4:

$$L_{perc} \;=\; \rho_{mon}\cdot V_4 \tag{3}$$

The observation (2) refers only to the percolating cluster.

Also, upon identification of the monopoles in the Abelian projection, one can measure the *non-Abelian* action associated with these monopoles. The results [8] turned in fact astonishing and can be explained only as a fine-tuning [9]. To explain the meaning of the fine tuning, let us remind the reader that the probability of finding any field configuration is a product of action- and entropy- factors. Then, it turns out that for the monopoles at the presently available lattices both factors diverge in the ultraviolet but cancel each other to a great extent:

$$P_{mon} = \exp(-S_{mon}) \times (\text{entropy}) \;\sim \exp(-c_1\cdot L/a)\cdot\exp(+c_2\cdot L/a)\,, \tag{4}$$

$$\frac{|c_1 - c_2|}{a} \;\sim\; \Lambda_{QCD}\,, \tag{5}$$

where P_{mon} is the probability to find the monopole trajectory of the length L, a is the lattice spacing and $c_{1,2}$ could in principle depend on the a as well but are treated as constants to describe the data on the available lattices (with smallest lattice spacing $a \sim 0.06$ fm from Ref. [8]).

The observation (4) implies also that the monopoles look point-like on the presently available lattices. Indeed, let us normalize the phenomenological monopole action for the lattice monopoles to the case of point-like monopoles. For the Dirac monopole the action would be:

$$S \;=\; L\cdot\frac{1}{8\pi}\int\limits_a^\infty \mathbf{B}^2 d^3r \;=\; \text{const.}\,\frac{L}{ae^2}\,, \tag{6}$$

where e is the electric charge and $\mathbf{B}$ is the magnetic field of the monopole, $B^2 \sim 1/e^2r^4$.

Thus, at presently available lattices the MAP monopoles are structure-less, or point-like objects. This observation immediately triggers further questions since usually one considers generic non-perturbative fluctuations as large-scale. We would not discuss such questions now but simply stick to the phenomenological observation that at the presently available lattices monopoles can be treated as geometrical objects, with no size. One can advocate then [9] the use of the polymer approach to the field theory, see, e. g., Ref. [10].

Our basic idea here is to pursue a geometrical language to describe monopoles, without any explicit use of Lagrangians like (1).

In Sect. II we consider the relation between the percolation theory and the monopole physics. In Sect. III we use the percolation theory to evaluate the spectrum of the finite-size monopole clusters. In Sect. IV the same finite-size clusters are described in terms of

the simplest vacuum loop in the polymer representation. In Sect.V we comment on the properties of the percolating cluster as representing the ground state of the system. In Sect.VI we discuss measurements which could clarify further the structure of the effective theory describing the monopoles. In Sect.VII we present conclusions.

It is worth mentioning that in a few cases we reproduce in some detail argumentation well known in other fields, in particular in the condense matter. And in this sense our presentation is not original in these cases. However, we believe that the justification of the use and application of the condense-matter techniques to non-perturbative fluctuations of the Yang-Mills fields is new.

II. MONOPOLE CLUSTERS AND PERCOLATION

In this section we will outline interconnections between properties of monopole clusters, percolation theory [11] and field theory. The aim is to motivate our basic assumptions (see subsection II F) and to apply later the percolation theory to the monopole clusters.

A. Monopole condensation in compact $U(1)$

A two-line theory of the monopole condensation was presented first in Ref. [12] for the case of the compact (or lattice) $U(1)$ theory. We will review the derivation here and dwell on its connection with percolation.

The role of the compactness of the $U(1)$ lattice theory is to ensure that the Dirac string does not cost any energy (for a review see, *e.g.*, Ref. [4]). That is why in Eq. (6) we take into account only the energy of the radial field. Nevertheless, the monopoles are infinitely heavy in the limit $a \to 0$ and, at first sight, this precludes condensation since the probability to find a monopole trajectory of the length L is suppressed as

$$\exp(-S) \;=\; \exp\left(-\frac{c}{e^2}\cdot\frac{L}{a}\right) . \tag{7}$$

Note that the constant c depends on details of the lattice regularization but can be found explicitly in any particular case.

What makes the condensation feasible, is an exponentially large enhancement factor due to the entropy. Namely, a trajectory of length L for a point-like monopole can be realized on a cubic lattice in $N_L = 7^{L/a}$ various ways. To evaluate the N_L let us notice that the monopole occupies the center of a cube and at each step the trajectory can be continued to an adjacent cube. In four dimensions there are 8 adjacent cubes. However, one of them is to be excluded from the counting since the monopole world-line is non-backtracking[1]. Thus the entropy factor is:

$$N_L \;=\; \exp\left(\ln 7\cdot\frac{L}{a}\right) , \tag{8}$$

[1] If a piece of the trajectory is covered in the both directions it is not observed on the lattice. Physically, this cancellation corresponds to the cancellation between monopole and anti-monopole.

and it cancels the suppression due to the action (7) if the coupling e^2 satisfies the condition of criticality:

$$e^2_{crit} \;=\; c/\ln 7 \;\approx\; 1 \tag{9}$$

where we quote the numerical value of e^2_{crit} for the Wilson action and cubic lattice. At e^2_{crit} any trajectory length L is allowed and the monopoles condense. This simple framework is verified within about one percent accuracy as far as the prediction of e^2_{crit} is concerned [13, 14].

One can say that the coupling e^2 of the compact $U(1)$ is to be fine-tuned to trigger the phase transition.

B. Relation to percolation

In fact the derivation of the preceding subsection can be viewed as an application of the percolation theory [11]. Moreover, one thinks in terms of the simplest percolation possible, that is, uncorrelated percolation.

Indeed, monopoles are observed as trajectories on the links of the dual lattice [2]. Postulate that the probability to occupy a link is given by:

$$p \;=\; \exp(-c/e^2)\;, \tag{10}$$

compare Eq. (7) at $L = a$. The probability that (uncorrelated) links form a trajectory of length L is then given by Eq. (7).

Formation of an infinitely long, or percolating cluster at a critical value $p = p_{crit}$ is a common feature of percolating systems. In our case,

$$p_{crit} \;=\; 1/7\;, \tag{11}$$

see Eq. (9).

It might worth mentioning that in text-books one would rather find $p_{crit} = 1/8$ since the non-backtracking feature of the monopole trajectory is specific for charged particles. From the theoretical point of view the non-backtracking is a manifestation of the monopole charge conservation. Another consequence of this conservation law is the closeness of the trajectories which has not been taken into account so far. As we shall see in Sect. IV the closeness of the trajectories brings in only a pre-exponential factor and can be ignored at the moment for this reason.

C. Relation to field theory

The derivation in Section II A implies that the monopole condensation occurs when the monopole action is ultraviolet divergent. On the other hand, the onset of the condensation in the standard field theoretical language corresponds to zero mass of the magnetically charged field ϕ. It is important to realize that this apparent mismatch between the two languages is not specific for the monopoles at all. Actually, there is a general kinematic relation (see, e.g., Ref. [15, 16]) between the physical mass of a scalar field m^2_{prop} and the mass M defined in terms of the (Euclidean) action S, $M \equiv S/L$:

$$m^2_{prop} \cdot a \;\approx\; C^2_m \cdot (M(a) - \frac{\ln 7}{a})\,, \quad C^2_m = 8\,. \tag{12}$$

It is m^2_{prop} that enters the propagator of a scalar particle $D(p^2)$,

$$D_{sc}(p^2) \sim \frac{1}{p^2 + m^2_{prop}}, \tag{13}$$

not $M^2(a)$. The factor $\ln 7$ in Eq. (12) is specific for the "monopole walk" (see the preceding subsection). The Eq. (12) establishes a connection between uncorrelated percolation and propagator of a free particle (in Euclidean space),

$$C^2_m\, m^2_{prop} a^2 \approx \ln(p_{crit}/p), \quad p_{crit} > p\,,$$

where we neglect, as usual, higher corrections in a.

It is also worth mentioning that the overall normalization of the propagator of a free particle in the polymer approach reads in the leading (in the lattice units a) order as follows:

$$\sum_{paths} \exp\Big\{-M(a) L_{path}\Big\} = C^2_m\, a^2\, D_{sc}(x_i, x_f)\,, \tag{14}$$

where $D_{scalar}(x, x')$ is the standard field-theoretical propagator of a scalar particle with mass (12) in the Euclidean space-time, L_{path} is the length of a particular path connecting the initial and final points $x_{i,f}$. Note that the action factor for a given path, $\exp(-M \cdot L_{path})$, is similar to the action of a polymer of length L, with the chemical potential replaced by the mass $M(a)$

Eq. (12) demonstrates that the propagating mass can be kept finite in the limit $a \to 0$ only at price of fine tuning. Moreover, Eq. (12) looks exactly the same as the standard radiative correction to the mass of a scalar particle. In more detail, the first term in the right-hand side is actually the magnetic field energy. The $\ln 7$ term, on the other hand, plays the role of a counter-term and this counter-term is calculable in our case!

D. Percolation and the quadratic divergence in field theory

Now we come to explain an important difference between phase transitions in case of uncorrelated percolation and in case of the monopoles. This difference might come as a kind of surprise since the transition point can be determined within the percolation theory with a good accuracy (see the discussion of the $U(1)$ case above).

The difference concerns properties of a percolating system and of the monopole vacuum in case of $p > p_{cr}$ and in the confining phase, respectively, If $p > p_{cr}$ the product of probability and entropy factors reduces to $\exp(+|\varepsilon| \cdot L)$, where ε can be arbitrarily small. Then, it is clear that the longest possible trajectory wins and the cluster fills in a finite fraction of the whole lattice. In the percolation-theory language the statement is that the fractal dimension at $p > p_{crit}$ coincides with the space dimension D:

$$(D_{fr})_{perc} = D\ ,\ p > p_{crit}\ , \tag{15}$$

see, *e.g.*, Ref. [17] and references therein. Note that in the language of field theory $p > p_{crit}$ corresponds to a tachyonic mass, $m^2_{prop} < 0$, see Eq. (12).

In case of monopoles, Eq (15) would correspond to

$$(\rho_{mon})_{perc} \sim a^{-3}\ . \tag{16}$$

Note that in the field theoretical language Eq. (16) would be interpreted as a power-like divergence well known in perturbation theory of charged scalar particles. Indeed, already on dimensional considerations one would conclude that:

$$\langle|\phi|^2\rangle_{perc} \sim a(\rho_{mon})_{perc} \sim a^{-2} , \tag{17}$$

(for a more careful derivation see Ref. [9]).

The behaviour (16) is in a sharp contrast with experimental data, see (2) and this is what we mean by the difference in the behaviour of a percolating system at $p > p_{cr}$ and of the monopoles in the confining phase. In other words, the fine tuning exhibited by the monopoles in the confining phase is a specific feature of the vacuum state of the lattice gauge theory.

E. Fine tuning and Coulomb-like interaction

Let us emphasize again that in the present note we treat the fine tuning of the monopoles as a pure observation and look for its implications. We are not in position at all to track its origin back to the non-Abelian Lagrangian which after all determines the results of all the lattice measurements. One of very few theoretical points which we can nevertheless add is that a long-range, Coulomb-like force is a necessary ingredient of a fine-tuned theory. Of course, in case of color by long distances we mean $r \gg a$ but $r \leq \Lambda_{QCD}^{-1}$.

To substantiate the point, let us consider the confining phase of the compact $U(1)$. The relative simplicity of this case is that the dynamics is explicitly known and, moreover, determined by the value of the electric charge e which can be tuned arbitrarily "by hand" (while in the non-Abelian case the coupling runs and is not under control in this sense).

Thus, one can introduce two scales in the confining phase of the compact $U(1)$ in the following way:

$$R_{UV} = a , \quad R_{IR} = \frac{a}{\epsilon} , \quad \epsilon \equiv (e^2 - e^2_{crit}) > 0 ; \quad \epsilon \ll 1 . \tag{18}$$

At first sight, we are in the same situation as in case of uncorrelated percolation since we introduced a tachyonic mass,

$$e^{-S} \sim e^{-\mu \cdot L} , \quad \mu \sim -\frac{\epsilon}{a} ,$$

and nothing can prevent development of a quadratic divergence, see discussion above.

On the other hand it is known from the lattice measurements (see, *e.g.*, Ref. [18] and references therein) that the IR scale does coexist with the UV one. And, indeed, there is the loophole in our logic that introducing the tachyonic mass would immediately result in a quadratic divergence. This would be true if the interaction were local. However, imagine that there are other monopoles at a distance of order R_{IR}. Then these monopoles can modify the Coulomb field of the monopole considered by order unit at $r \sim R_{IR}$. This, in turn, can be interpreted as a change in the mass (due to interactions):

$$|\delta M(a)| \sim \frac{\epsilon}{a}. \tag{19}$$

This change can compensate for the "short-distance" tachyonic mass so that the particles are kept away from each other at a distance of order R_{IR}. Which also means that there is

no power-like UV divergences (see discussion above). This double-face interpretation of (19) as IR and UV effects is a unique feature of the Coulomb-like interaction.

Note that we are not really giving a proof that the introduction of R_{IR} is self-consistent. We just argue that *without* long-range forces the fine tuning would be not possible at all. In case of non-Abelian theories there are many more open questions since the monopole action is not bounded from below (see, *e.g.*, Ref. [4]) and the fact that the action is tuned to the entropy is even more difficult to explain theoretically.

F. Fine tuning and $\lambda\phi^4$ models

It is worth to emphasize that the fine tuning which is observed for the lattice monopoles is of the same generic type considered so mystifying in case of the Standard Model. There is, however, a peculiarity in our case. Indeed, if we compare the "natural" estimate for ρ_{mon} (16) with the data (2) we see that there are three powers of a^{-1} that are "tuned away", not two powers as in the standard field-theoretical language. And, indeed, the data (2) imply in the "naive" $a \to 0$ limit [9]:

$$\begin{aligned} \lim_{a\to 0}\langle|\phi|^2\rangle \;&\sim\; \rho_{mon}\cdot a \;\to 0\,, \\ \lim_{a\to 0} m^2_{prop} \;\sim \lim_{a\to 0}\frac{1}{a}(M(a) - \ln 7/a) \;&\sim\; \frac{\Lambda_{QCD}}{a} \;\to\; \infty \\ \lim_{a\to 0} m^2_{gluon} \;&\sim\; g^2\langle|\phi|^2\rangle \;\to\; 0\;, \end{aligned} \tag{20}$$

where m_{gluon} is the (dual) gluon mass which arises in theories of the type (1).

Eqs. (20) are of course not what one would expect for the standard $\lambda\phi^4$ theory with spontaneous symmetry breaking. It is worth emphasizing, however, that the limit $a \to 0$ in Eqs. (20) should be understood with some reservations. What we mean actually in Eqs. (20) by $a \to 0$ limit is "the lattice spacing a as small as possible within availability on present lattices". The behaviour (20) can actually change in the academic limit $a \to 0$ (i.e. at lattice spacings much smaller than those presently available). Subsequent considerations in the present paper would actually suggest such a possibility (see Sect. V).

Although Eqs. (20) may not survive at smaller a, they do answer questions why new point-like fluctuations (implied by the fine-tuning) do not disturb the standard β-function of the $SU(2)$ gluodynamics at existing lattices. Indeed, according to (20) all the scalar degrees of freedom are actually removed from the physical spectrum if $a \to 0$.

The properties (20) imply also that the potential energy $V(|\phi|^2)$ scales in the limit $a \to$ and our effective theory can well be a useful approximation to study the vacuum properties. In particular, the monopole confining mechanism survives in the limit [9] $a \to 0$.

G. Formulating the main hypothesis

After all these preliminary discussions we are set to formulate our main hypothesis. Namely, we will assume that we can consider the point

$$p \;=\; p_{crit} \tag{21}$$

as adequately describing the physics in the *confining phase* of the non-Abelian gauge theory. The justification for this hypothesis is the fine tuning observed on the lattice and discussed in length above.

There are important reservations to be made. Namely, the fine tuning does imply that the physics is so to say "frozen" at $p = p_{cr}$ as far as the UV scale is concerned. However, as far as the dependence on the scale Λ_{QCD} is concerned it could be different than at the point (21). Moreover, we can give a more quantitative meaning to the scale "Λ_{QCD}" in the case considered. The point is that the percolating cluster has self-crossings and the length of the trajectory between these self-crossings can be considered as the monopole free-path length. Direct measurements show [19] that this length (measured along the trajectory) scales:

$$L_{free} \approx 1.6\ fm\,. \tag{22}$$

Thus, we can apply our hypothesis as long as

$$L_{free} \gg L \gg a\,. \tag{23}$$

In the next section we exploit the percolation theory to describe the structure of the finite-size monopole clusters satisfying (23).

III. FINITE-SIZE CLUSTERS AND PERCOLATION

A. Data and percolation picture

Detailed data on the structure of the monopole clusters were obtained in [7]. As was mentioned above, there is a single percolating cluster, whose length grows with the lattice volume, and finite-size clusters. In this section we will concentrate on the finite-size clusters satisfying the condition (23). These clusters are characterized, first of all, by their length. It was found that the length spectrum is described by a power law:

$$N(L) = \frac{c_4}{L^\tau} \cdot V_4\ , \tag{24}$$

where

$$\tau \approx 3 \tag{25}$$

for all lattice spacings and sizes tested and the coefficient c_4 depends only weakly on β. For our purposes we can neglect this dependence.

Another important characteristics of the clusters is their radius R_L as function of the length L. By the radius one understands the average distance between two cluster links:

$$R_L^2 = \frac{a}{L}\sum_{i=1}^{L/a}(\mathbf{x}_i - \bar{\mathbf{x}})^2 = \frac{a^2}{2L^2}\sum_{i,j=1}^{l/a}|\mathbf{x}_i - \mathbf{x}_j|^2\ , \tag{26}$$

where $\mathbf{x}_{i,j}$ are coordinates of the links and $\bar{\mathbf{x}} = (1/L)\sum \mathbf{x}_i$. The measurements indicate:

$$R_L \sim \mathrm{const}_1 + \mathrm{const}_2\sqrt{L}\,. \tag{27}$$

Thus, our problem is to clarify whether the data (25) and (27) can be understood within the percolation theory (and our main hypothesis, see subsection II F). It is encouraging to observe that even without any dynamical input we can conclude that the lattice data on

the finite-size monopole clusters reveal a picture typical for percolating systems. Indeed, a generic form of the spectrum, for $p < p_{crit}$ is

$$N(L) \sim \frac{\exp(-\mu \cdot L)}{L^{\tau}} \cdot V_4 \,, \tag{28}$$

where τ is the so called Fischer exponent and μ vanishes at the critical point, $p = p_{crit}$. The latter is easy to understand since at the point of the phase transition the correlation length is infinite and there is no dimensional parameter left. Moreover, a power law like (27) is common to the percolating systems,

$$R_L \sim L^{1/D_{fr}} \,, \tag{29}$$

and D_{fr} is called the fractal dimension. For the monopoles we apparently have from the lattice measurements:

$$(D_{fr})_{mon} \approx 2 \,. \tag{30}$$

Thus, the data on the finite-size monopole clusters exhibit a typical percolation picture and the next question is whether it is possible to evaluate the exponents (25) and (30).

B. Hyperscaling relation

Similarity between percolation and properties of dynamical systems undergoing the phase transition is well known [11]. However, direct evaluation of the critical exponents requires, generally speaking, a particular dynamical input on the system considered. Mostly, knowledge on the excitation spectrum is required.

However, there is a general relation between the critical exponents which we are going to apply first. Note that our considerations here are not specific for the monopoles and similar, *e.g.*, to the analysis of Ref. [20]. Still, for the sake of orientation let us mention a few points important for the application of the percolation theory to the monopoles clusters.

First, one derives all relations for $p \leq p_{cr}$, without considering $p > p_{cr}$. As we discussed in length in Sect. II, uncorrelated percolation can be indeed relevant to the monopole physics only at $p \leq p_{cr}$. Second, all the relations of the percolation theory follow more or less directly from the assumption (28) and it is worth emphasizing that the spectrum (28) is very natural physics-wise. Indeed, the presence of the exponential factor at $p - p_{cr} < 0$ is obvious from considerations of subsection II B. Moreover at the point $p = p_{cr}$ there is no scale left and the power like dependence of the spectrum on L is the only dependence allowed. In this way one comes to include the factor $L^{-\tau}$ as well.

Starting from the spectrum (28) it is quite straightforward, see, *e.g.*, Ref. [20], to derive the following well-known relation:

$$\tau = \frac{D}{D_{fr}} + 1 \quad , \tag{31}$$

where d is the dimension of space-time and D_{fr} is in fact $D_{fr}(p = p_{cr})$.

In our case,

$$\tau \approx 3 \,, \quad D \equiv 4, \quad D_{fr} \approx 2 \,. \tag{32}$$

Thus, Eq. (31) is satisfied within the error bars of the lattice measurements and this observation is one of our main results. In view of its importance, we will later rederive (32) and some generalizations of it in the language of field theory.

C. Fractal dimension

As is mentioned above, the relation between the length and radius of the cluster is determined by the fractal dimension, see Eq. (27). The fractal dimension, in turn, is determined by the kind of the walk.

In fact we are dealing with the *monopole walk* which, to the best of our knowledge, has not been studied in detail in the literature. However, the characteristics of the monopole walk are so to say flanked by the characteristics of the well known *random* and *self-avoiding* walks. Indeed, the monopole walk chooses freely one of 7 directions available at each step. This is in common with the random walk. On the other hand, choosing the eighth direction would result in an immediate self-crossing and this is forbidden. The latter feature is in common with the self-avoiding walk. However, in contradistinction from the self-avoiding walk, self-crossings are allowed for the monopole trajectories at later stages.

The observation central for this section is that in $D = 4$ the fractal dimension $D_{fr} = 2$ both for the random and self-avoiding walks. Therefore we can predict $D_{fr} = 2$ for the monopole walk as well.

In more detail, for the random walk one has in any number of space dimensions [2]:

$$(D_{fr})_{random} \;=\; 2\,. \tag{33}$$

For the self-avoiding walk one has (the so called Flory's fractal dimension):

$$(D_{fr})_{self-avoiding} \;=\; \frac{D+2}{3} \tag{34}$$

which is valid for $D \leq 4$ and gives exactly the same $D_{fr} = 2$ at $D = 4$.

Thus, the experimental value $(D_{fr})_{mon} = 2$ appears to be well understood theoretically.

IV. SEEING FREE MONOPOLES AT SHORT DISTANCES

From our discussion of the relation between the uncorrelated percolation and free-field theory (see Sect. II C) it is clear that the results of the preceding section can be derived in terms of Feynman graphs as well [3]. Moreover, since we are considering relatively short trajectories, see (23), the effect of the mass can be neglected. In this section we will reproduce the results above by considering the simplest Feynman graph relevant, that is a single vacuum loop. This approach allows us to derive also some generalizations, like length-of-the-trajectory spectrum at finite temperature.

A. Polymer representation for a massless particle propagator

In this subsection we will evaluate the Fischer exponent τ (see (31)) starting from the observation that the fluctuations of the field ϕ are massless on the scale a. To this end we reproduce first the basic points of the polymer approach to the field theory of a free scalar particle, see, *e.g.*, Refs. [15, 16, 21, 22].

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[2] This relation, in connection with the monopole clusters, is in fact mentioned in Ref. [7].

[3] We are grateful to D. Diakonov for a discussion on this point and providing us with a reference to [21].

The partition function for a closed polymer is:

$$Z = \int \mathrm{d}^4 x \sum_{N=1}^{\infty} \frac{1}{N} e^{-M\cdot N} Z_N(x,x)\,, \tag{35}$$

where M is the chemical potential and $Z_N(x_0, x_f)$ is the partition function of a polymer broken into N segments:

$$Z_N(x_0,x_f) = \Big[\prod_{i=1}^{N-1} \int \mathrm{d}^4 x_i\Big] \prod_{i=1}^{N} \Big[\frac{\delta(|x_i - x_{i-1}| - a)}{2\pi^2 a^3}\Big] \tag{36}$$

This partition function (35) contains a summation over all atoms of the polymer weighted by the Boltzmann factors. The δ–functions in (36) ensure that each bond in the polymer has length a. The starting point of the polymer is x_0 and the ending point is $x_f \equiv x_N$.

Note that there is a pre-exponential factor $1/N$ in Eq. (35). This is due to considering closed trajectories. Indeed, the factor is introduced to compensate for the N-multiple counting of the same closed trajectory in the partition function (35) since any atom on this trajectory can be considered as the initial and final point. As we shall see later, our final result crucially depends on the pre-exponential factors.

The crucial step to relate (36) to a free particle path integral is the so called coarse–graining. Namely, the N–sized polymer is divided into m units by n atoms ($N = mn$), and the limit is considered when both m and n are large while a and $\sqrt{n}a$ are small. We get,

$$\prod_{i=\nu n}^{(\nu+1)n-1} \frac{1}{2\pi^2 a^3}\delta(|x_i - x_{i+1}| - a) \to \Big(\frac{2}{\pi n a^2}\Big)^2 \exp\Big\{-\frac{2}{n\,a^2}(x_{(\nu+1)n} - x_{\nu n})^2\Big\}\,, \tag{37}$$

where the index i, $i = \nu n \cdots (\nu+1)n-1$, labels the atoms in ν^{th} unit. The polymer partition function becomes [22]:

$$\begin{aligned} Z_N(x_0,x_f) \;=\; \text{const}\cdot \Big[\prod_{\nu=1}^{m-1} \mathrm{d}^4 x\Big] \Big[\Big(\frac{2}{\pi n a^2}\Big)^{2m} \exp\Big\{-2\sum_{\nu=1}^{m} \frac{(x_\nu - x_{\nu-1})^2}{n a^2}\Big\}\Big] \\ \cdot \exp\Big\{-\sum_{\nu=1}^{m} n\cdot a\,\mu\Big\}\,. \end{aligned} \tag{38}$$

The x_i's have been re-labeled so that x_ν is the average value of x in at the coarser cell. Note also that at this stage there appears the chemical potential μ related to the original parameter M through

$$\mu = M - \ln 7/a\,, \tag{39}$$

as is discussed above.

Using the variables:

$$s = \frac{1}{8} n a^2 \nu\,, \quad l = \frac{1}{8} a^2 N\,, \quad m^2_{prop} = \frac{8\mu}{a}\,, \tag{40}$$

one can rewrite the partition function (35) as

$$Z = \text{const}\cdot \int_0^{\infty} \frac{\mathrm{d}l}{l} \int_{x(0)=x(l)=x} Dx\, \exp\Big\{-\int_0^{l} \Big[\frac{1}{4}\dot{x}^2_\mu(s) + m^2_{prop}\Big]\,\mathrm{d}s\Big\}\,. \tag{41}$$

Note that the mass renormalization in Eq. (40) is consistent with Eqs.(12,39).

After these preliminary steps we can readily derive the distribution in the length of the trajectories in the massless case. To this end, let us rescale x_μ and s in such a way, that there is no l dependence left in the action if $m_{prop} = 0$:

$$L = l/a, \quad \tilde{s} = s/l, \; \tilde{x}_\mu = x_\mu/\sqrt{l} \; . \tag{42}$$

Then, indeed,

$$Z = \text{const} \cdot \int_0^\infty \frac{\mathrm{d}L}{L} \cdot \; I \; , \tag{43}$$

where

$$I \equiv \int\limits_{\tilde{x}(0)=\tilde{x}(l)=\tilde{x}} D\tilde{x} \; \exp\Big\{ -\frac{1}{4} \int_0^1 \dot{\tilde{x}}_\mu^2(\tilde{s}) \, \mathrm{d}\tilde{s} \Big\} . \tag{44}$$

At first sight, Eq. (43) implies that we have a dL/L spectrum since there is no L dependence left otherwise. However, the actual spectrum refers to the number of loops in a unit volume (see (24) and the discussion of it). Thus, the dL/L spectrum refers to the volume in the $\tilde{x}$ units. Since the $\tilde{x}^D \sim x^D L^{-D/2}$ we have in the physical-volume units:

$$N(L)dL = c_D \; V_D \; \frac{dL}{L^{1+D/2}} \; , \tag{45}$$

where V_D is the volume in D-dimensional space and c_D is a constant. Although we are interested only in the $D = 4$ case we kept D as variable to emphasize that we rederive in fact the hyperscaling relation (31). Note that $D_{fr} = 2$ is implicit in our derivation and is encoded in fact in the transformation (37).

B. Coulomb-like interaction

So far we considered approximation of free particles which corresponds to the dominance of the monopole self-energy. We expect, however, that the monopoles interact also Coulomb-like. Other, effective interactions are not ruled out either. The Coulomb-like interaction can readily be included into the action in the polymer representation. The corresponding extra piece in the action is given by:

$$S_{Coulomb} = \frac{g_M^2}{2} \int_0^l \int_0^l ds_1 ds_2 \dot{x}_{1,\mu} D_{\mu\nu}(x_1 - x_2) \dot{x}_{2,\nu} \tag{46}$$

where g_M is the magnetic charge, $D_{\mu\nu}(z)$ is the massless photon propagator, and the dot, as usual, means differentiation with respect to the proper time.

The action (46) is manifestly invariant under the rescaling (42):

$$S_{Coulomb} = \frac{g_M^2}{2} \int_0^1 \int_0^1 d\tilde{s}_1 d\tilde{s}_2 \dot{\tilde{x}}_{1,\mu} D_{\mu\nu}(\tilde{x}_1 - \tilde{x}_2) \dot{\tilde{x}}_{2,\nu} \; , \tag{47}$$

and, therefore, the $1/L^3$ behaviour of the spectrum (45) should be still true for[4] $D = 4$ upon inclusion of this interaction[5].

C. Finite temperatures, finite volume lattices

Formalism of Section IV A can readily be generalized to the case of a non-zero temperature. Indeed, we should rewrite now in terms of a length-of-the-trajectory distribution the propagator of a massless particle at finite temperature.

As usual, finite temperature in Euclidean space corresponds to a compactified fourth direction. Thus, we simply write the L-distribution corresponding to an ensemble of non-interacting particles with masses $m^2 = (2\pi n)^2 T^2$ in d=3 space–time:

$$N(L) \;=\; c_3\, V_3\, \frac{1}{L^{5/2}} \sum_{n\in\mathbb{Z}} \exp\,(\,-\,(2\pi n T)^2 L\cdot a\,)\,, \tag{48}$$

where $V_3 = V_4 T$ is the three dimensional volume of the time–slice. Note that in the limit $T \to 0$ the sum over the exponentials is proportional to $1/(T\sqrt{L\cdot a})$ and we come back to (45) upon proper normalization of the constant c_3,

$$c_3 \;=\; 2\, c_4\, \sqrt{\pi a}\,,$$

where c_4 enters Eq. (24). However, in the opposite limit, $T \to \infty$ we effectively get three dimensional theory with the volume V_3 and the loop distribution $N(L) \propto L^{-5/2}$.

A remark on the range of validity of (48) is now in order. The point is that in (48) we disregard chemical potential $\mu \neq 0$. Whether this is a valid approximation depends on the numerical values of the parameters involved, μ, T, a. In particular, if we tend $a \to 0$ and keep μ in physical units the approximation (48) is not valid.

Proceeding in a similar way we can derive also the spectrum on a finite-size lattice of the volume $V_4 = \prod_{\mu=1}^{4} X_\mu$, where X_μ is the length of the space in $\mu^{\rm th}$ direction. Assuming the periodic boundary condition we get:

$$N(L) \;=\; \frac{c_0}{L} \prod_{\mu=1}^{4} \sum_{n_\mu\in\mathbb{Z}} \exp\,\{\,-\,(2\pi n_\mu/X_\mu)^2 L\cdot a\}\,, \tag{49}$$

where

$$c_0 \;=\; 16\pi^2\, c_4 a^2\,.$$

V. INFRARED CLUSTER AND PROPERTIES OF THE GROUND STATE

In this section we discuss geometrical properties of the percolating cluster. It was suggested already in [15, 22] that the percolating cluster corresponds to the ϕ-field condensate

[4] The case $D = 4$ is special because the gauge coupling is dimensionless.

[5] We do not discuss here subtleness which can arise from consideration of the coinciding points, $x_1 \to x_2$, in Eq. (46).

in the classical approximation [6]. However, it is only the phenomenon of the fine tuning that makes the properties of the vacuum state non-trivial. Indeed, we have now two coexisting scales, a and Λ_{QCD}^{-1}. Moreover, we will see that the properties of the percolating cluster differ radically from the properties of the finite-size clusters.

A. Lattice data

We have already mentioned that the density of the monopoles in the percolating cluster scales, see (2). Recently, further measurements on the geometrical elements of the percolating cluster have been performed [19]. In more detail, the cluster consists of self-crossings and segments connecting the crossings. It was found that the average distance between the crossings measured along the trajectory scales, see Eq. (22). Violations of the scaling in L_{free} are negligible for all the lattices tested.

One can also measure the average of the shortest, or Euclidean distance, $\langle d \rangle$, between the two crossings connected by segments. In this case violations of the scaling are more significant and, roughly, the data can be approximated as:

$$\langle d \rangle \sqrt{\sigma} \;\approx\; (0.65 \;+\; [(a\sqrt{\sigma}) - 0.25]) \;, \tag{50}$$

where a is the lattice spacing, σ is the string tension for the $SU(2)$ gluodynamics, and $0.15 \;<\; a\sqrt{\sigma} \;< 0.35$.

Thus, we are confronted with the problem of interpreting the data (2), (22), (50). The first step is to reduce the set of data to a simple picture: normalized to a free-particle case the measurements correspond to an infinite mass [19], $m_{prop}^2 \sim a^{-2}$. Which is at first sight a shocking observation defying everything that we said so far. Postponing discussion of the physics till the next subsection let us reiterate the reasons for such an interpretation of the data.

For a free particle, one can measure its mass by comparing the Euclidean distance between two points with the length along the corresponding trajectory connecting the same points. Indeed, one obtains the distance L by differentiating the partition function (35) with respect to the chemical potential μ defined in Eq. (39). On the other hand, the dependence of a free particle propagator on the chemical potential enters through the factor $\exp(-m_{prop}\, d)$ where the m_{prop} is as in Eq. (12). Differentiating with respect to μ the propagator $D(m_{prop}, d)$ of a free particle of mass m_{prop} we get therefore:

$$\langle L_{free\ particle} \rangle \;= -\frac{\partial}{\partial \mu} \ln D(m_{prop}, d) \;\approx\; \frac{d_{free\ particle}}{2(m_{prop}(a) \cdot a)} \;, \tag{51}$$

where we neglected higher corrections in a and m_{prop} (generally speaking, m_{prop} also depends on a).

Two particular cases are worth mentioning. First, if m_{phys}^2 is in the physical units, $m_{prop}^2 \sim \Lambda_{QCD}^2$ then we would have

$$\langle L \rangle \;\sim\; \frac{\langle d \rangle}{a}$$

[6] Literally, a model with a tachyonic mass was introduced first in [22] and such a model would result in $\rho_{mon} \sim a^{-3}$.

which is in blatant disagreement with the data (50). Moreover if we assume cancellation of only the leading $1/a$ term in (12) we would expect:

$$\langle L \rangle \ \sim \frac{\langle d \rangle}{\sqrt{a}} \ ,$$

which still cannot be reconciled with the data either.

Finally, if $m^2_{prop} \sim a^{-2}$ then

$$\langle L \rangle \ \sim \ \langle d \rangle$$

which does agree with the data (50) for a available as far as we neglect the correction linear in a.

To summarize, we are coming to a paradox. Indeed, even if we accepted $m^2_{prop} \sim a^{-2}$ to satisfy $\langle L \rangle \sim \langle d \rangle$ this would not help since we would not be able then to explain the persistence of the physical scale in (22), (50).

B. Analogy to the Mößbauer effect

Thus, we see that the finite-size clusters and the percolating cluster exhibit different patterns of the monopole kinematics. In the former case we could neglect the monopole mass while in the latter case it is not possible at all. Looking for an analogy, we naturally come to the Mößbauer effect. Indeed, one could say that the effect is the difference in kinematics inherent to the decays of a free atom and of an atom belonging to a lattice in its ground state. Formally, the kinematics of the decay of the atom belonging to the lattice looks as if the atom had an infinite mass. Thus, the analogy we are going to pursue is between the percolating monopole cluster and atoms in the ground state of a lattice of atoms. The reason for the analogy is that in the both cases we are dealing with ground states of systems with a gap.

Let us first remind the reader the basic features of the Mößbauer phenomenon (for a review see, *e.g.*, Ref. [23]). One considers decay of an atom belonging to a lattice of atoms. Then there exists a probability P_0 that the lattice is not excited at all, i.e. no phonons are emitted. Then the recoil momentum is transferred to the whole of the lattice and, clearly, this corresponds to an infinite effective mass of the decaying atom.

In the language of the quantum mechanics, the probability of the Mößbauer transition is determined by the following matrix element:

$$P_0 \ = \ |\langle i| \exp(i \mathbf{p}_\gamma \cdot \mathbf{X}_{atom}) |i\rangle|^2 \tag{52}$$

where $|i\rangle$ is the ground state of the lattice, $\mathbf{X}_{atom}$ is the position of the decaying atom and $\mathbf{p}_\gamma$ its recoil momentum. Then, there are two crucial quantities which determine the estimate of P_0, namely the "naive" recoil energy R and the energy of the gap. For non relativistic kinematics,

$$R \ = \ \mathbf{p}^2_\gamma / 2 M_{atom} \ \ ,$$

and the gap energy, ω_{gap}. A rough estimate for P_0 is as follows:

$$P_0 \ \sim \exp(-R/\omega_{gap}) \ . \tag{53}$$

Thus, when the recoil energy is much larger than the gap, $R \gg \omega_{gap}$ P_0 tends to zero and kinematics is the same as for a free-atom decay. In this limit we would come back to the parton-like picture.

The lattice data on the monopoles indicate that it is the opposite limit, $R \ll \omega_{gap}$, which can be relevant to the monopoles (if the analogy is correct at all).

C. Quantum propagator as Brownian motion

The Mößbauer effect concerns kinematics in the Minkowski space. On the other hand, we are considering the monopole motions in the Euclidean space. Moreover, there are no decays of the monopoles of course and, at first sight, there is no physical quantity analogous to $\mathbf{p}_\gamma$. However, we will argue in this subsection that the famous Brownian picture [11] for the quantum propagator provides a key to identify an analog to $|\mathbf{p}_\gamma|$ in the monopole case.

The classical Brownian motion (see, *e.g.*, Ref. [11]) is the motion of a particle of mass M_{Br} which results from exchange of momenta with particles of a medium which are not observed and chaotic. Then the Brownian motion is a random walk with the step

$$b \ \sim \ \frac{\delta p \cdot \delta \tau}{M_{br}} \ ,$$

where $\delta\tau$ is the average time between collisions and δp is the average momentum transfer during the collision.

In case of the quantum propagator one should rather think in terms of the Brownian motion of a particle attached to a string with a non-vanishing tension. This is the origin of the inertia, or of the mass term (for details see Sect. 4.1). As for a non-vanishing Δp it is now quantum in origin and can be estimated from the uncertainty principle. Since the monopole trajectory is measured on the scale a, generically $\Delta p \sim a^{-1}$ and is parametrically large if $a \to 0$.

This simplest estimate should be corrected, however, for the fact that one can introduce mass and kinetic energy only after coarse-graining see Sect. 4.1. Therefore,

$$\Delta p \ \sim \frac{1}{a \cdot \sqrt{n}} \tag{54}$$

where n was introduced in Eq. (37). Note that n does not depend on a and for practical estimates one can use

$$\sqrt{n} \ \approx (2 \div 3) \ .$$

D. Limit $a \to 0$: academic and realistic

Coming back to our analogy with the Mößbauer effect, we conclude that in the limit $a \to 0$ we expect that the existence of the gap is not important and we can think in terms of the parton model. Indeed, in the relativistic kinematics, $R \sim \Delta p \sim a^{-1}$ while the mass of "free monopole" $m_{prop} \sim a^{-1/2}$ and can be neglected compared to Δp at small a. Moreover, the gap is provided by the glueball mass, m_{gl} and does not depend on a at all.

This conclusion could be foreseen: at short distances, i.e. in the limit $a \to 0$ one can neglect the effects of the binding of the monopoles. What is more surprising is that for realistic values of a we are in fact far from the academic limit $a \to 0$. This is due to interplay of various numerical factors. Consider, for example, $a = (3 \text{ GeV})^{-1}$ which corresponds to smallest a available. Then:

$$m_0^2 \ \sim \ \frac{8\mu}{a} \ \sim \ 3 \ \text{GeV}^2, \ \ (\Delta p)^2 \ \sim \ 2 \ \text{GeV}^2 \tag{55}$$

where we used $n \approx 5$ (see Eq. (54)) and the chemical potential (39) can be estimated with the help of Eq. (22) as $\mu \sim (1.6\ fm)^{-1}$. Thus the ordering of m_{prop} and Δp is still reversed at the presently available lattices compared to the expectations in the $a \to 0$ limit.

The onset of the parton model is expected when copious excitation of glueballs is favored. Since the glueball is relatively heavy, $m_{gl} \sim 1.5$ GeV the realistic values of Δp are far from satisfying the condition

$$(\Delta p)^2 \ \gg \ (m_{prop} + m_{gl})^2 \ . \tag{56}$$

Thus, we come to the following conclusion:

The recoil-free geometrical picture of the percolating cluster is perfectly consistent with the quantum mechanics as far as the resolution (which is of order $\sqrt{n}a$) is not too high. At better resolution we expect to loose both point-like monopoles and the geometrical picture itself.

Actually, in the data of Ref. [8] one can already see some hints on violations of the $1/a$ behaviour of the monopole field-theoretical mass $M(a)$. Thus, we are inclined to correlate these first indications to appearance of the monopole structure [8] and the scaling violations in d_{av}, see Eq. (50). Our prediction is that these indications become clearer at smaller a.

One can roughly estimate the values of a_{crit} at which the parton picture begins to prevail over the recoil-less kinematics as:

$$a_{crit}^{-1} \ \sim \ (10 \div 30)\text{GeV} \ . \tag{57}$$

At such a one can also expect that the monopoles do not look point-like. Indeed, in the field theoretical language it is quite trivial that the dominance of the inelastic processes and appearance of the form-factor for an "elastic" process are determined by the same dynamics.

Appearance of such a large mass scale as (57) might come as a surprise. It is worth noticing therefore that there exists accumulating, although indirect evidence on the relevance of large mass scales to the QCD phenomenology (see, *e.g.*, Ref. [25] and references therein). Now we see how such a scale can be build up on Λ_{QCD}, proceeding through the glueball mass and other numerical factors.

VI. MONOPOLES AND GLUEBALLS

Thus, the next crucial question seems to be the coupling of the monopoles to glueballs. In the most general way the monopole-glueball interaction can be studied by measuring the correlation function:

$$K(x-y) \ = \ \langle 0|\rho(x), \rho(y)|0\rangle \ , \tag{58}$$

where

$$\rho(t) \equiv \phi^+(\mathbf{x}, t)\phi(\mathbf{x}, t) \ .$$

Asymptotic at large distances is sensitive to a massive excitation [24]:

$$\lim_{t\to\infty} \langle 0|\rho(t), \rho(0)|0\rangle \ = \ c_1 \ + c_2\, e^{-m_{gl}\cdot t} , \tag{59}$$

which is likely to be a glueball.

As far as we approximate monopoles as point-like *and* confine ourselves to an effective $\lambda\phi^4$ theory *without gluons* then correlators like (58) are given by a sum over closed monopole loops:

$$K(x-y) \ = \ \frac{\partial^2}{\partial j_1 \partial j_2} Z(M) \ \Big|_{j_1=j_2=0} \tag{60}$$

where

$$M(z) \;=\; j_1\delta^4(x-z) + j_2\delta^4(y-z)\;,$$

and the partition function is given by:

$$Z(M) \;=\; \frac{1}{4\pi^2}\int d^4x_0 \int_0^\infty \frac{dL}{L^3} e^{-m^2 La} \langle e^{-V(x(\tau))} \exp\,(\,-\int M(x(\tau))d\tau)\rangle_{x_0}\,, \qquad (61)$$

where $\langle ...\rangle_{x_0}$ means averaging over all the closed paths. Further details and definitions can be found, *e.g.*, in the review [26]. What is important for us now is that the sum over the paths can be calculated numerically using the lattice data since the monopole trajectories are directly observed.

The constant in Eq. (59) is related to the ρ_{mon}:

$$c_1 \;=\; (8\rho_{mon}\cdot a)^2\,, \qquad (62)$$

and is entirely determined by the percolating cluster. Determination of the glueball mass would require, on the other hand, sensitivity to the quantum corrections, or to the finite-size clusters[7].

It could quite well be so that keeping the closed paths alone would not be adequate to determine the glueball mass through (59). This would be a signal that gluonic intermediate states are important and the theory is not unitary without inclusion of such states. Since the gluons are not detected directly on the lattice the pure gluonic intermediate states would look as breaks in the monopole loops.

Thus, measurements of the correlator (58) would be very important to further understand the structure of the effective theory of the monopole interactions.

VII. CONCLUSIONS

We started with the picture according to which non-Abelian gluodynamics, when projected onto the scalar-field theory via monopoles, corresponds to a fine tuned theory [9]. The monopoles which we considered are defined ("detected") through the Maximal Abelian projection. However, the mass scales which exhibit mass hierarchy are gauge independent. The scales are provided by the $SU(2)$ invariant action per unit length of the monopole trajectory, on one hand, and by inverse "free path length" (see (23)), on the other. The observed fine tuning suggests the use of the geometrical language to describe the monopoles.

In this paper we confronted the polymer approach to the effective theories of the magnetically charged fields with the lattice data on the monopole clusters.

There are two types of clusters, namely, finite-size and percolating. In the former case we have demonstrated that the length spectrum and the dependence of the size of the clusters on their length are well understood within the percolation theory. An alternative language is the simplest vacuum loop in the polymer language.

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[7] The percolating cluster alone at distances $d \gg d_{av}$ is representing a cluster of fractal dimension $D_{fr} = 4$ (see our discussion of the uncorrelated percolation in Sect. II) and cannot be sensitive to the glueball mass.

There is a striking difference, however, between the finite-size clusters and the percolating cluster. In the former case one thinks in terms of the vanishing mass of the monopoles (on the scale of the lattice spacing a):

$$(m^2_{mon})_{finite-size} \approx 0 \cdot a^{-2}. \tag{63}$$

This picture is confirmed by evaluation of the critical exponents just mentioned. On the other hand, the straightforward interpretation of the geometry of the percolating cluster leads to [19]:

$$(m^2_{mon})_{percolating} \sim a^{-2}. \tag{64}$$

We have argued that the apparent discrepancy between the effective monopole masses extracted in the two ways is naturally resolved if one takes into account that the finite-size clusters correspond to quantum corrections while the percolating cluster corresponds to the ground state. The quantum corrections reveal then presence of a massless (on the scale of a) excitation. In case of the ground state, we observe an analog of the Mößbauer effect. The onset of a parton-like description is delayed numerically by some factors, most notably by a relatively high glueball mass m_{gl} where m_{gl} plays the role of the gap.

We also predict that at much smaller a (see (57)) one should see the convergence of (64) to (63).

Thus, first applications of the polymer picture to the monopole clusters turn successful. However, justification for such an approach remains at present pure phenomenological in nature and relies on the observation of the fine tuning. Any further theoretical development is therefore to be checked by measurements on the lattice. In particular, studies at smaller a, both theoretical and numerical, are desirable.

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